Debt Capacity, Optimal Capital Structure, and Capital Budgeting Analysis

Hai Hong and Alfred Rappaport

Hai Hong is Senior Lecturer in Business Administration at the University of Singapore. Alfred Rappaport is Professor of Accounting and Information Systems at Northwestern University.

■ As far back as 1955, Solomon [9] recognized that capital budgeting decisions should consider the debt-carrying capacity that a proposed capital project adds to the firm. Since then, much work has been done on the nature of debt capacity and its value to the firm. The value of carrying debt was quantified by Modigliani and Miller (MM) [7] in the context of a competitive market with corporate taxes and no insolvency costs. Lewellen [5] further clarified the analysis of debt capacity by considering portfolio effects of an acquisition whose cash flows are less than perfectly correlated with those of the acquiring firm. Building on the MM framework and the capital asset pricing model, Bower and Jenks [1] explicitly incorporated the tax benefits of debt capacity in setting discount rates for individual projects. Martin and Scott [6] extended the Bower-Jenks analysis by expressing debt capacity in terms of a target probability risk of insolvency for the firm.

The capital budgeting literature cited does not deal explicitly with the firm's optimal capital structure. This paper seeks to do this and to analyze the implications of optimal structure in capital budgeting decisions.

We argue that the debt capacity of a firm should be expressed as the amount of debt at the firm's optimal capital structure, *i.e.*, its optimal debt level. This contrasts with frequently recommended approaches for determining debt capacity such as target debt ratios [1, 8] and target probabilities of insolvency [5, 6]. A logical implication of our approach is that the debt capacity contribution of a proposed capital investment project is the increase in the firm's optimal debt level resulting from the investment. After the amount of the debt capacity contribution is determined, there remains the problem of ascertaining its *value* to the firm. In contrast to other methods based on MM [1, 6], we estimate the value of debt capacity not as the tax rate multiplied by the debt capacity, but at a somewhat lesser amount because of the accompanying insolvency cost.

Debt Capacity and Optimal Capital Structure

MM show that in a world with taxes and no bankruptcy costs the value of a firm is given by

$$V = \frac{X(1-T)}{k_S} + \frac{Tk_D D}{k_D}\ , \qquad (1)$$

where X is the expected annual before-tax operating cash flows (assumed constant forever); T = the firm's tax rate; k_S = the rate at which the market capitalizes the after-tax cash flows of an unlevered company in the risk class; D = market value of debt (assumed permanently outstanding); k_D = rate at which the market capitalizes interest payments on debt. The MM equation is an application of the value additivity principle [3]. V is the added value of two streams — the uncertain stream X, capitalized at k_S, and the tax shelter stream $Tk_D D$, capitalized at k_D. In the idealized MM world, firm value is maximized when the firm is financed totally by debt. In practice, of course, creditors will not permit the firm to borrow without limit. More important, the firm implicitly assigns a cost to the risk of insolvency and trades off this cost against the tax benefit of debt so as to attain optimal capital structure. This is, of course, a well-established proposition [2, 10] that has been formalized in a one-period state preference framework by Kraus and Litzenberger [4].

Following Lewellen [5], Martin-Scott [6], and others, we define insolvency as a state in which the firm's operating cash flows cannot meet contractual debt obligations, *i.e.*, $X < k_D D$. Failure of the firm to extricate itself from insolvency by selling claims to future cash flows results in bankruptcy, a more serious form of financial distress. For convenience we call the total risk of financial distress simply insolvency risk. Accordingly, the cost imputed to insolvency risk is termed insolvency cost (C). Optimal capital structure is attained when the marginal tax benefit of debt is exactly offset by the marginal insolvency cost. Without any loss of generality, we may write C in the multiplicative form $C = Dk_I$, where k_I is the average insolvency cost per unit of debt. We should expect k_I to be an increasing function of D and to depend on the statistical distribution of the annual operating cash flows:

$$k_I = k_I(D \mid X, \sigma),$$

where X is the mean and σ the standard deviation of the cash flows ($\tilde{X}$).

Invoking again the value additivity principle, we may rewrite the MM Equation (1) as

$$V = \frac{X(1-T)}{k_S} + TD - Dk_I. \qquad (2)$$

Thus the MM formulation (Equation 1) overvalues the firm by the insolvency cost, Dk_I. Note that we can rewrite Equation (2) in its equivalent form using the average cost of capital, k_a [2, 10]:

$$V = \frac{X(1-T)}{k_a}. \qquad (2a)$$

Substituting $X(1-T) = k_a V$ from Equation (2a) into Equation (2) and rearranging terms:

$$k_a = k_s\,[1 - \frac{D}{V}(T - k_I)]. \qquad (3)$$

If we ignore insolvency costs, then k_a is understated, and consequently V is overvalued as noted above.

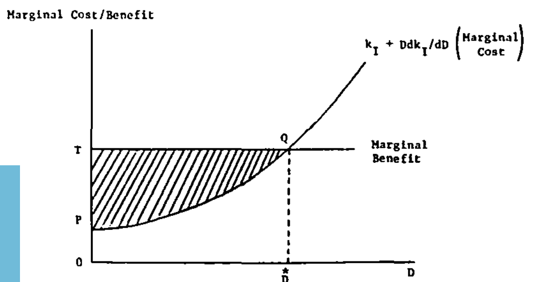
Returning to Equation (2), we determine optimal capital structure by maximizing V while holding X constant and varying D. V is at a maximum when $dV/dD = 0$. Differentiating Equation (2) with respect to D.

$$\frac{dV}{dD} = T - [k_I + D\frac{dk_I}{dD}] = 0. \qquad (4)$$

In Equation (4), T is the marginal benefit and $(k_I + Ddk_I/dD)$ the marginal cost of an additional dollar of debt. Exhibit 1 illustrates the marginal relationship between insolvency cost and tax benefits, and the optimal debt level $\overset{*}{D}$ (the debt capacity of the firm).

In this illustration, convexity of Dk_I ensures that the marginal cost function $k_I + Ddk_I/dD$ is upward sloping. The gross tax benefit of debt is $T\overset{*}{D}$ (area of the rectangle $OTQ\overset{*}{D}$). The insolvency cost for debt capacity $\overset{*}{D}$ is $\overset{*}{D}k_I$ (area $OPQ\overset{*}{D}$ under the marginal cost curve). The cross-hatched area PTQ therefore gives the net benefit of carrying debt, *i.e.*, area PTQ = $\overset{*}{D}[T - k_I]$. When the firm is at its optimum capital structure.

Exhibit 1. Marginal Cost and Benefit of Carrying Debt



$$V = \frac{X(1-T)}{k_S} + T\overset{*}{D} - k_I\overset{*}{D}. \quad (5)$$

Insolvency Cost Function

While the preceding development relies only on the reasonable assumption that total insolvency is convex in D, it may be instructive to specify a functional form for k_I. Our aim in specifying the functional form is to illustrate the procedure used in estimating insolvency costs and optimal capital structure. For this purpose, we choose a somewhat simple functional form with some intuitive appeal:

$$k_I = cP(\tilde{X} \leq Dk_D), \quad (6)$$

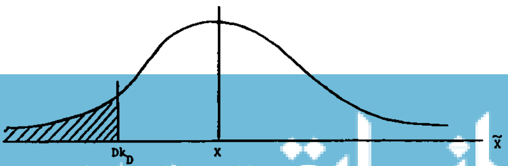
where c is a scaling constant, and P stands for probability. The equation says that average insolvency cost is proportional to the probability risk of insolvency $P(\tilde{X} \leq Dk_D)$. The probability function $P(\tilde{X} \leq Dk_D)$ is given by the cross-hatched area in Exhibit 2. If $\tilde{X}$ has a normal distribution, we may rewrite k_I as

$$k_I = cN[-(X - Dk_D)/\sigma], \quad (7)$$

where N[•] is the cumulative normal probability function.

The parameter c incorporates a large number of factors, including liquidation costs, goodwill lost in defaulted payments, and the appropriate rate of discount for these costs. The value of c will vary from one industry to another and, to a lesser extent, across companies within the same industry. It may be imputed to any given company by presuming that it is at optimal capital structure. In other words, estimate $P(\tilde{X} \leq Dk_D)$ from past company data, and determine the value of c such that its present capital structure is optimal by our criterion. Different assets may well have different c's. The c of a firm could then be the weighted average of the c's of its divisions or assets. We should emphasize, of course, that the functional form we have postulated above undoubtedly oversimplifies the complicated interaction of factors associated with bankruptcy and financial distress. A more thorough specification of k_I is the subject of future research.

Exhibit 2.



As we should expect, a change in X or σ will change the functional form of k_I. For example, if X becomes larger or σ smaller, the risk of insolvency is reduced for a given amount of D, and the marginal cost schedule in Exhibit 1 would shift to the right. This means that for any given amount of debt the marginal cost per unit of debt would decrease, and the firm would have increased its total debt capacity. Such would be the result of investing in a new project, which we consider next.

Debt Capacity and Capital Budgeting

Consider a firm A that proposes to invest in a project B. Let D_A = the market value of A's debt; X_A = the expected annual before-tax operating cash flows for A; X_B = the expected annual before-tax operating cash flows contributed by B; and σ_A and σ_B = the standard deviations associated with X_A and X_B, respectively. Assuming A already has optimal capital structure, the value of the firm V_A is given by

$$V_A = \frac{X_A(1-T)}{k_{SA}} + T\overset{*}{D}_A - k_I\overset{*}{D}_A, \quad (8)$$

where k_{SA} is the rate at which the market capitalizes an unlevered company in the same risk class as A, and $\overset{*}{D}_A$ is its debt capacity.

The addition of cash flows from B will change A's insolvency cost function k_I. Let k_I' be its insolvency cost function after acquiring B. Similarly, let primes (') denote quantities after B has been added. The insolvency cost is now $D'k'_I$, where $k_I' = k_I'(D' | X', \sigma_A)$. A new optimal debt level is attained at $\overset{*}{D}'$ as the marginal cost curve shifts to the right (Exhibit 3). The debt capacity of firm A increases by $(\overset{*}{D}_A' - \overset{*}{D}_A)$, and its net benefit from this increased capacity is the cross-hatched area in Exhibit 3.

The value of firm A increases to

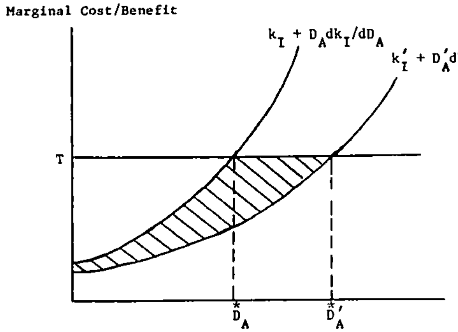
$$V_A' = \frac{X_A(1-T)}{k_{SA}} + \frac{X_B(1-T)}{k_{SB}} + T\overset{*}{D}_A' - k_I'\overset{*}{D}_A', \quad (9)$$

where k_{SB} is the rate at which A capitalizes the after-tax cash flows of B. From Equations (8) and (9) the value of B to firm A is obtained:

$$V_A' - V_A = \frac{X_B(1-T)}{k_{SB}} + [T(\overset{*}{D}_A' - \overset{*}{D}_A) - (\overset{*}{D}_A' k_I' - \overset{*}{D}_A k_I)]. \quad (10)$$

The last expression in brackets is the cross-hatched area in Exhibit 3, the valuation of B's debt capacity contribution to A. This is in contrast to Bower-Jenks [1] and Martin-Scott [6] who value the contribution at

Exhibit 3. Optimal Debt Capacity Before and After Asset Acquisition



the tax rate T multiplied by the incremental debt capacity. Such an estimate will tend to overvalue the debt capacity because it ignores insolvency cost. Thus, while these authors recognize the existence of insolvency by assuming a target debt level, they do not subtract the insolvency cost from the tax benefits of debt to determine the net value of debt capacity.

Numerical Example

Let
$$X_A = 120{,}000$$
$$\sigma_A = 80{,}000$$
$$c = 5.0$$
$$T = 0.50$$
$$k_D = 0.08$$
$$k_{SA} = 0.12$$
$$X_B = 20{,}000$$
$$\sigma_B = 15{,}000$$
$$k_{SB} = 0.10$$

Correlation coefficient between $\tilde{X}_A$ and $\tilde{X}_B = 0.50$

In this example we assume that k_D is constant as the amount of debt changes. In practice, k_D may in fact increase with the debt ratio. For typical debt ratios of nonfinancial companies at optimal capital structures, the change in k_D is likely to be relatively small and to have little impact on insolvency cost (see appendix). For convenience we ignore this complication in our example.

The value of 5.0 for c is arbitrary. It does yield, however, a probability of insolvency of around 8%, which we believe to be reasonable. If c were much larger, indicating substantially greater expected insolvency costs, a much lower probability of insolvency would obtain at optimal capital structure. Conversely, a much smaller value of c would imply an unduly high risk of insolvency.

From Equation (7),

$$k_I = 5.0\,N\left[\frac{-(120{,}000 - 0.08D_A)}{80{,}000}\right].$$

At the optimal capital structure for firm A

$$T = 0.5 = k_I + D_A \frac{dk_I}{dD_A}.$$

Solving (see appendix), we find that $\overset{*}{D}_A = 110{,}000$, $k_I = 0.4115$, and the risk of insolvency is $N(-1.39)$ or 8.2%. From Equation (8), the value of the firm $V_A = 120{,}000(1-0.5)/0.12 + (0.5 \times 110{,}000) - (0.4115 \times 110{,}000) = 509{,}735$.

With the addition of B,

$$\sigma_A' = [80{,}000^2 + 2(0.5)(80{,}000)(15{,}000) + 15{,}000^2]^{1/2}$$
$$= 88{,}500$$
$$X_A' = 140{,}000.$$

For optimal capital structure,

$$T = 0.5 = k_I + D_A \frac{dk_I'}{dD_A'}.$$

Solving (see appendix), $\overset{*}{D}_A' = 179{,}100$, $k_I' = 0.3890$, and the risk of insolvency is $N(-1.42)$ or 7.8%. Thus the contribution of B to the debt capacity of A is 69,100. The net value of this additional capacity is [see Equation (10)]:

$$T\,[\overset{*}{D}_A' - \overset{*}{D}_A] - (\overset{*}{D}_A' k_I' - \overset{*}{D}_A k_I)$$
$$= 0.5(179{,}100 - 110{,}000) - (69{,}670 - 45{,}265)$$
$$= 34{,}500 - 24{,}405$$
$$= 10{,}095$$

Note that this is considerably smaller than the gross (tax) benefit of 34,500 that would be obtained if insolvency cost is ignored. The value of B to firm A is $20{,}000(1-0.5)/(0.10) + 10{,}095 = 110{,}095$.

The risk of insolvency of the firm at its optimal capital structure has declined slightly (from 8.2% to 7.8%) as a result of the acquisition. Thus attaining optimal capital structure is not generally the same as maintaining some target risk of insolvency.

We have assumed a relatively small correlation coefficient of 0.5 between the cash flows of A and B. If the correlation were larger, the standard deviation σ_A' would have been larger and the contribution to debt capacity correspondingly smaller. This is consistent with Lewellen's observation that acquisitions that are negatively correlated with the firm contribute more to debt capacity than those that are highly positively correlated, *i.e.*, diversification enhances debt capacity [5].

Commentary

In capital budgeting analysis, it is important to estimate 1) the amount of debt capacity contributed by the proposed project; and 2) the value of this debt capacity to the firm. Previous studies have typically estimated debt capacity by assuming a target debt level specified either by a given debt ratio or by requiring that the firm not exceed some probability risk of insolvency. We have extended these approaches by relating debt capacity directly to capital structure, thereby integrating capital budgeting analysis with the optimum financing paradigm. Furthermore, we show that, having determined the amount of debt capacity contributed by a project, it is not correct to then value the debt capacity only in terms of its tax benefits to the firm. Rather, debt capacity brings with it added insolvency costs that our computational procedure shows can materially offset the tax benefits.

In this paper we have, for purely illustrative purposes, used a simple insolvency cost function that is linear in the probability of insolvency. While this yields reasonable results, we feel that much more work needs to be done on alternative insolvency cost functions. Such functions would incorporate more precisely the actual costs of insolvency and the risk attitudes of the management, shareholders, and creditors of the firm. It should also be possible to estimate these functions empirically using data on capital structures of firms.

Appendix.

Solving for $\overset{*}{D}_A$ and $\overset{*}{D}'_A$

$$\frac{d}{dD_A}[D_A k_I] = k_I + D_A \frac{dk_I}{dD_A} = 0.5$$

Let $-(120{,}000 - 0.08D_A)/80{,}000 = y$

$$0.5 = 5.0\,[N(y) + D_A n(y) \frac{dy}{dD_A}]$$

$$= 5.0\,[N(y) + D_A \times \frac{0.08}{80{,}000} n(y)] \qquad \text{(A-1)}$$

where $N(\cdot)$ is the cumulative normal function and $n(\cdot)$ is the normal density function.

By trial and error, or by graphing the right-hand side of Equation (A-1), we find that $y = -1.39$, $\overset{*}{D}_A = 110{,}000$, risk of insolvency = $N(-1.39)$, or 8.2%, and $k_I = 5.0N(-1.39) = 0.4115$. The RHS of Equation (A-1) is $5.0\,(0.0823 + 110{,}000 \times 0.08 \times 0.1518/80{,}000) = 5.0\,(0.0823 + 0.0167) = 0.495 \simeq 0.5$. Similarly, to determine $\overset{*}{D}'_A$:

$$0.5 = 5.0\,[N(y') - D'_A n(y') \times \frac{0.08}{88{,}500}]$$

where $y' = \dfrac{-(140{,}000 - 0.08D'_A)}{88{,}500}$

By trial and error,

$y' = -1.42$, $k'_I = 0.3890$

$\overset{*}{D}'_A = 179{,}100$ and risk of insolvency $N(-1.42) = 7.8\%$.

In this example we have assumed that k_D is constant, *i.e.*, $dk_D/dD_A = 0$. In practice, the cost of debt will tend to rise with the level of debt, *i.e.*, $dk_D/dD \geq 0$. To take this into account, we rewrite Equation (A-1) as

$$0.5 = 5.0\,[N(y) + \frac{D_A n(y)\,(k_D + D_A dk_D/dD_A)}{80{,}000}]$$

$$= 5.0\,[N(y) + \frac{D_A n(y) k_D}{80{,}000}\,(1 + \frac{D_A}{k_D}\,\frac{dk_D}{dD_A})]$$

Ignoring dk_D/dD_A is clearly a good approximation if $\frac{D_A}{k_D}\,\frac{dk_D}{dD_A} << 1$, *i.e.*, if the elasticity of k_D with respect to D_A is much less than 1. Even if this elasticity were as large as 1, however, its impact on the RHS of Equation (A-1) is not extremely large since the $N(y)$ term dominates the RHS expression.

References

1. Richard S. Bower and Jeffrey M. Jenks, "Divisional Screening Rates," *Financial Management* (Autumn 1975), pp. 42–49.
2. Eugene F. Brigham, *Financial Management*, Hinsdale, Ill., Dryden Press, 1977.
3. Charles W. Haley and Lawrence D. Schall, *The Theory of Financial Decisions*, New York, McGraw-Hill Book Co., 1973.
4. Alan Kraus and Robert H. Litzenberger, "A State-Preference Model of Optimal Financial Leverage," *Journal of Finance* (September 1973), pp. 911–22.
5. Wilbur G. Lewellen, "A Pure Financial Rationale for the Conglomerate Merger," *Journal of Finance* (May 1971), pp. 521–37.
6. John D. Martin and David F. Scott, Jr., "Debt Capacity and the Capital Budgeting Decision," *Financial Management* (Summer 1976), pp. 7–14.
7. Franco Modigliani and Merton H. Miller, "Corporate Income Taxes and the Cost of Capital: A Correction," *American Economic Review* (June 1963), pp. 433–43.
8. Stewart C. Myers, "Interactions of Corporate Financing and Investment Decisions — Implications for Capital Budgeting," *Journal of Finance* (March 1974), pp. 1–25.
9. Erza Solomon, "Measuring a Company's Cost of Capital," *Journal of Business* (October 1955), pp. 95–117.
10. James C. Van Horne, *Financial Management and Policy*, 4th ed., Englewood Cliffs, N.J., Prentice-Hall, Inc., 1977.